Extension of Methods Used in Analyzing Building Thermal Loads

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ABSTRACT

This paper details analytical procedures for use in building thermal loads analysis and modeling. The first technique discussed details the mathematical basis of a recursion relationship developed to define building/space weighting factors. The weighting factors generated encompass radiative and conductive heat gain and space air temperature fluctuations. To preclude the use of weighting factors, an additional technique is presented which makes possible the calculation of either a pseudo or real surface temperature; thus, space loads can be ascertained directly. Finally, the third procedure relates the frequency of specific input excitations to overall building response and/or energy consumption; this is primarily a procedural technique for simplifying the analysis and selection of architectural alternatives.

KEYWORDS

Building Simulation
Thermal Loads
Design Methods and Tools

INTRODUCTION

Space and/or building thermal loads and temperatures can be defined either through the solution of thermal balance equations such as those used in NBSLD[4] and BLAST[5] or by convolution of the heat gain or loss excitations of the building with a set of weighting factors (DOE-2[6]). These two methods of calculating the thermal load have proven to be equivalent if the weighting factors are defined for the specific space being analyzed. The DOE-2 computer program is a total energy analysis routine. Calculation of building loads is performed in addition to simulations of air handling systems and central energy plants. For the purpose of computational efficiency, the DOE-2 program was structured to calculate load distributions through the use of weighting factors. Previously, a limited number of tabulated average weighting factors were available for the energy analysis; however, these weighting factors did not reflect the specific nature of the space being analyzed. A method has now been developed by Consultants Computation Bureau in which it is possible to ascertain space specific weighting factors due to conductive and radiative heat gains and losses, as well as air temperature fluctuations. The next section of this paper details

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the analytical basis of a thermal load recursion relationship which describes the response of a space to a unit pulse excitation. Space specific weighting factors can be derived from this recursion relationship.

An alternative method of determining thermal loads is through the separation of the convective and radiative heat transfer coefficients. By accomplishing such a breakdown of the combined coefficients, surface temperature calculations can be made and, thus, space loads ascertained directly. A discussion is presented of this technique, which, in the case of DOE-2, provides an alternative scheme for generating the load components at constant space temperature.

The weighting factor method provides for the component breakdown of the input excitations. This ability, in conjunction with the Z-transform definition of weighting factors, indicates that a frequency domain approach can be used to relate input quantities to output loads for each component separately. Initial results from such a frequency response analysis technique are presented. The weighting factor/Z-transform is related to the Laplace S-transform so that amplitude ratio and phase shift relationships are established for a continuous frequency range. Use of this analysis technique provides considerable insight into the influence that individual design parameters have on the load response of a structure to various environmental and occupancy related load excitations.

WEIGHTING FACTOR RECURSION METHOD

This section presents a discussion of the analytical derivation of space specific weighting factors using a recursion technique. The weighting factors are generated through a convolution scheme relating the load response of a space to a unit excitation of radiation and/or space temperature variation. A heat balance is performed on both the inside and outside surfaces of the walls enclosing a space. The resultant solution temperatures yield a space load at time t=0. All subsequent load values are related to the initial value through the recursion method.

The surface heat balance matrix can be written as:

$$[C]\,[T] = [B] \tag{1}$$

where for i less than or equal to the number of inside surfaces (NS), the inside balance terms are:

$$\begin{aligned} C_{i,i} &= A_i[X_i(0)+HI_i+\sum_{m=1}^{NS} G_{i,m}] \\ C_{i,m} &= -A_iG_{i,m} = C_{m,i} = -A_mG_{m,i} \\ B_i &= e_i(t) - \phi_i(t) \end{aligned} \tag{2}$$

with

$$\begin{aligned} e_i(t) &= A_iHI_iTR(t) + RI_i(t) \\ \phi_i(t) &= A_i[\sum_{j=1}^{t} (X_i(j)TI_i(t-j)-Y_i(j)TO_i(t-j))] \end{aligned} \tag{3}$$

For $(NS+1) \leq i \leq 2NS$, the outside balance yields:

$$C_{i,i} = A_i[Z_i(0)+HO_i]$$

$$C_{i,m} = -A_iY_i(0) = C_{m,i} = -A_mY_m(0) \tag{4}$$

$$B_i = e_i(t) - \phi_i(t)$$

with

$$e_i(t) = A_iHO_iTOA(t) + RO_i(t)$$

$$\phi_i(t) = A_i[\sum_{j=1}^{t}(Z_i(j)TO_i(t-j)-Y_i(j)TI_i(t-j))] \tag{5}$$

Using [k] to represent the inverse matrix of [C], the solution temperatures are found by solving the set of resulting simultaneous equations. The development of a recursion procedure follows from the fact that at time t=0, with $\phi(0)=0$, the temperatures are strictly a function of the excitation:

$$TI_i(0) = \sum_{n=1}^{2NS} k_{i,n}e_n(0) \tag{6}$$

For all times other than t=0, both the external and internal excitations for a unit pulse input are zero, thus:

$$TI_i(t) = -\sum_{n=1}^{2NS} k_{i,n}\phi_n(t) \tag{7}$$

Since $\phi(t)$ and therefore TI(t) are related to the past temperature values, it is apparent that the space load for t=0 is related to its past values when one considers:

$$Q(t) = \sum_{i=1}^{NS} Q_i(t) = \sum_{i=1}^{NS} A_iH_i \sum_{n=1}^{2NS} k_{i,n}\phi_n(t) \tag{8}$$

The capability of giving unit input excitations to the external surfaces should be noted since it provides a relatively easy scheme to study inter-zone heat transfer.

Consultants Computation Bureau compared the results from general purpose weighting factor and thermal balance techniques capable of accepting the same input excitations. In addition, the thermal balance scheme was used to define the weighting factors for use in the weighting factor method. Figure 1 presents a diagram depicting the nature of the calculation procedure used for the comparison. The thermal balance technique performs a heat balance at both inside and outside surfaces of a space defined primarily by its structural properties. Since the algorithm accepts any arbitrary input excitation form in flux or temperature, coordinate orientation is unnecessary and the only geometrical considerations result from the view factors which are calculated in an approximate manner through the area ratio relationship.[1] A matrix solution is used to obtain inside surface temperatures from which the space load or air temperature is obtained. Initially, the space properties are defined, after which a unit pulse in radiation and air temperature is applied to determine the weighting factors through the use of the recursion relationship. The next step entails the application of a sinusoidal radiation or other arbitrary radiation input at fixed space temperature. A direct comparison of the space load calculation is then made. In the thermal balance procedure, this same excitation is again input and the space temperature allowed to fluctuate. The weighting factor procedure, however, uses the constant temperature load profile in conjunction with the previously defined space air weighting factors to calculate the temperature variation.

For the radiation pulse case, sets of 2, 3 and 4 weighting factors were generated by visual determination of succeeding common ratios. The load response to a 10,000 BTU amplitude sinusoidal radiation input for both the thermal balance and weighting factor methods is shown in Figure 2. Readily apparent is the convergence of two schemes with increased number of factors. A more exact procedure for determining the common ratios would no doubt improve the comparative results. Figure 3 shows the space air temperature variation for the two methods. The maximum difference is roughly 1/2°F. Presented on Figure 4 is another temperature variation. In this case, the excitation arises from a 10°F sinusoidal variation in outside air temperature and a 10K BTU radiation input. The constant space temperature load generated by the thermal balance was used as input to the variable temperature weighting factor calculations.

These results indicate that there are no substantial differences between loads and temperatures calculated by either the weighting factor or thermal balance methods of approach. As stated in Reference 1, whatever differences exist are due to the program specific aspects of the routines defining the input quantities necessary for solution.

PSEUDO SURFACE TEMPERATURE

An analytical method is described in this section in which an average inside surface temperature is calculated. In DOE-2, this procedure negates the use of either the radiation or conduction weighting factors. In addition, this technique represents an alternative approach to the thermal balance matrix solution method of space load determination.

The method employs the basic concept of heat balance at both the inside and outside surfaces. Additionally, the combined inside surface radiative/convective heat gain/loss components are split such that:

$$qr_i(t) = A_i HR_i [TA_i(t) - TI_i(t)] \tag{9}$$

and

$$qc_i(t) = A_i HC_i [TR - TI_i(t)] \tag{10}$$

The area-averaged temperature, $TA_i(t)$, can be approximated by:

$$TA_i(t) = \left(\sum_{m=1}^{NS-1} A_m TI_m(t-1)\right) / \left(\sum_{m=1}^{NS-1} A_m\right) \tag{11}$$

for $m \neq i$, where NS is the number of surfaces within the space. This equation reflects the influence of the other surface temperatures on surface i at time t resulting from the radiative coupling of the surfaces. The radiative heat transfer coefficient would be

$$HR_i = 4E_i \sigma [TA_i(t)]^3 \tag{12}$$

Solving the heat balance equations simultaneously for inside surface temperatures allows the immediate calculation of the space load.

FREQUENCY RESPONSE METHODS

Weighting factors are generated by application of a distributed unit flux or temperature pulse from which a time response in space load is produced. An efficient means of interpreting architectural parameter influences on resulting weighting factors and load response can be obtained by application of frequency response methods of analysis. This section details the transformation from a discrete Z-plane to a continuous S-plane from which the amplitude ratio and phase shift characteristics of the load response can be ascertained for a sinusoidal excitation input.

The space load response to an arbitrary flux input can be expressed in the form of a Z-transform as follows:

$$Q(Z) = [(V_0+V_1Z^{-1})/(1+W_1Z^{-1})]\ q(Z) \qquad (13)$$

where the V's and the W represent the weighting factors from previously computed load response to a unit pulse input. Application of a bilinear transformation, through the use of a new variable R, such that:

$$Z = (1+R)/(1-R) \text{ with } R= j\nu \qquad (14)$$

enables the Z-transform to be conveniently analyzed in the ν complex plane. The R-transform of Eq. 13 would be:

$$Q(R) = K_R[(R+a)/(R+b)]\ q(R) \qquad (15)$$

with

$$\begin{aligned} K_R &= V_0[(1-V_1/V_0)/(1-W_1)] \\ a &= (1+V_1/V_0)/(1-V_1/V_0) \\ b &= (1+W_1)/(1-W_1) \end{aligned} \qquad (16)$$

A Bode diagram approach characterizing the amplitude ratio and phase shift can be used to define the varying frequency response for a sinusoidal input function, $q(t)=\sin \nu t$. Simplification of the analysis results when one employs the well-known asymptotic relationships for $\nu=0$ and $\nu=\infty$. Application of these techniques to Eq. 16 results in the following amplitude ratio values:

$$\text{for } \nu=0 \qquad M = 1.0$$

$$\text{for } \nu=\infty \qquad M = [2V_0-(1-W_1)]/1-W_1 \qquad (17)$$

Using these asymptotes and realizing that break frequency positions would be at $\nu=a$ and $\nu=b$ (see Eq. 15), a generalized Bode diagram can be constructed from which the real frequency response can be obtained by application of a 3 decibel correction at each breakpoint.

Figure 5 presents such a composite and amplitude ratio plot relating values of V_0 and W_1 to the frequency and amplitude ratio. The common ratio or W_1 value can be directly related to the frequency since the pole position or (b) root in Eq. 16 is strictly dependent on this value. The second breakpoint or (a) root is obtained from the intersection of the V_0 and W_1 curves. Figure 6 shows the corresponding composite asymptotic phase shift diagram.

To realize the application of these curves, consider Figure 7, which presents the time response of both a lightweight and heavyweight structure to a unit sinusoidal radiation input with a period of 24 hours. The weighting factors used to generate the response are also listed on the figure.

To preclude the need for such a response presentation, one could use the procedure shown on Figures 8 and 9 to obtain essentially the same information without resorting to a time response generation scheme. Figure 8 presents the amplitude ratio resulting from application of the techniques discussed herein, and Figure 9 illustrates the phase shift relationship.

By utilizing these methods, a direct comparison can be made of the relative amplitude ratio and phase shift difference one can expect from varying sets of weighting factors and, thus, structural characteristics.

CONCLUSIONS

The current public domain thermal loads simulation program, DOE-2, uses weighting factors to define the thermal characteristics of buildings due to conductive and radiative excitations. Whereas, in the past, pretabulated weighting factors were used, a simple recursion formula has been developed which relates the building/space response to a unit input excitation from which building/space specific weighting factors can be generated. Subsequent analysis has shown that using weighting factors defined for a particular space yields loads and temperatures equivalent to a thermal balance matrix solution.

To preclude the use of weighting factors, an additional technique is presented which makes possible the calculation of either a pseudo or real surface temperature. This technique considers the non-linear radiation exchange effects; is computationally more efficient than the matrix solution of the detailed thermal balance method and allows the consideration of the multizone cases.

The dynamic response of buildings can best be described through the use of special analysis, which is a frequency domain approach to system analysis. A method is described herein which relates the frequency of specific input excitations to overall building response and/or energy consumption. Knowledge of the dominant frequency spectrum and resultant energy utilization can lead to optimal building design procedures.

NOMENCLATURE

A - surface area
B - heat balance excitation vector
C - heat balance time independent matrix
e - heat balance excitation component
E - emissivity
G - heat exchange factor between surfaces
H - inside/outside heat transfer coefficient
HC - inside surface convective heat transfer coefficient
HI - inside surface heat transfer coefficient
HO - outside surface heat transfer coefficient
HR - inside surface radiation heat transfer coefficient
M - amplitude ratio (output/input)
Q - space load
qc - convective heat gain component
qr - radiative heat gain component
RI - inside heat flux component
RO - outside heat flux component
t - time
T - heat balance temperature
TA - inside surface area average temperature
TI - inside surface temperature
TO - outside surface temperature
TOA - outside air temperature
TR - space air temperature
X, Y, Z - surface response factors
V, W - space weighting factors
ϕ - heat flux coupling between inside/outside surfaces
σ - Stefan-Boltzmann constant

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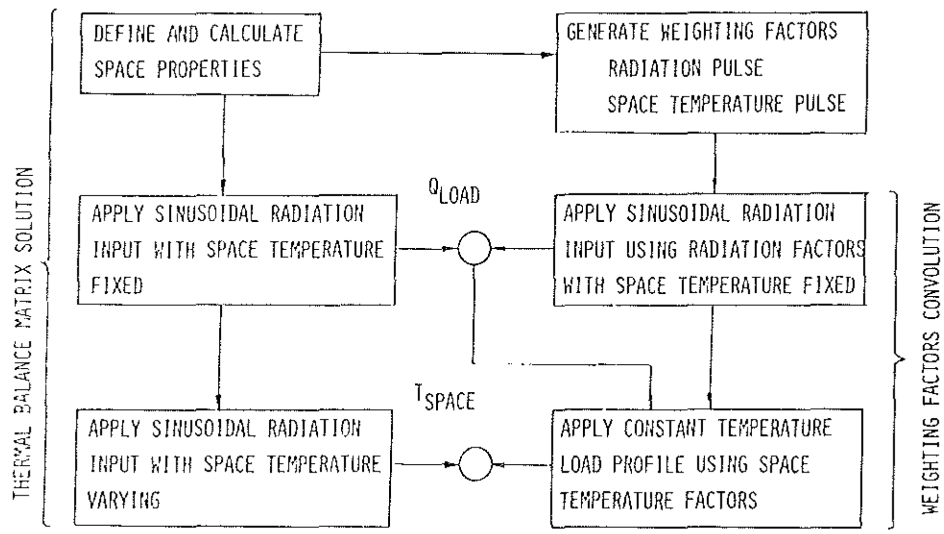


Fig. 1. Weighting Factor/Thermal Balance Comparison Flow Diagram

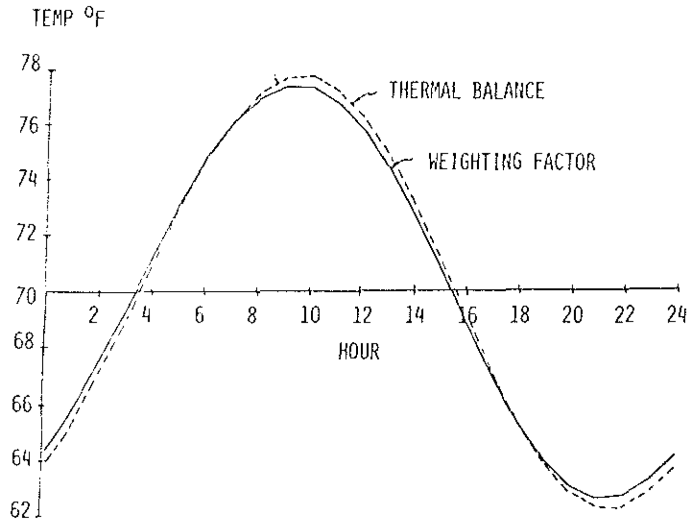
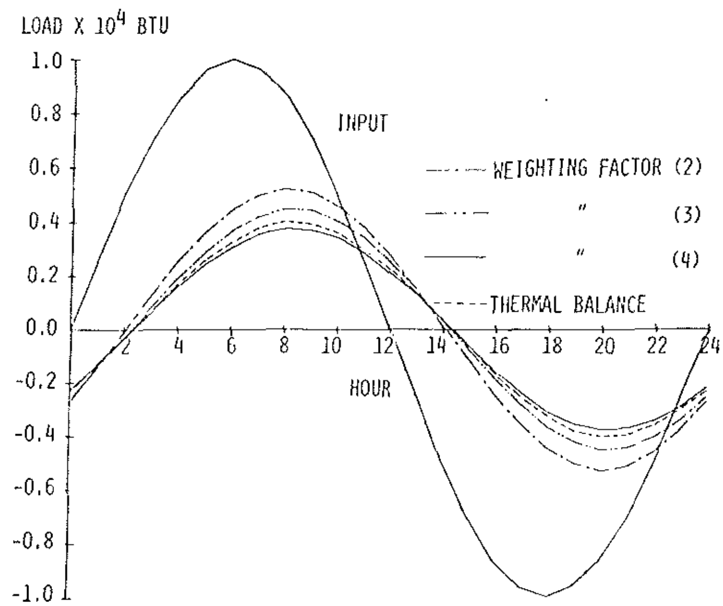


Fig. 2.
Weighting Factor/Thermal Balance Load Response Comparison for a 10K BTU Sinusoidal Radiation Input

Fig. 3.
Weighting Factor/Thermal Balance Space Air Temperature Response Comparison for 10K BTU



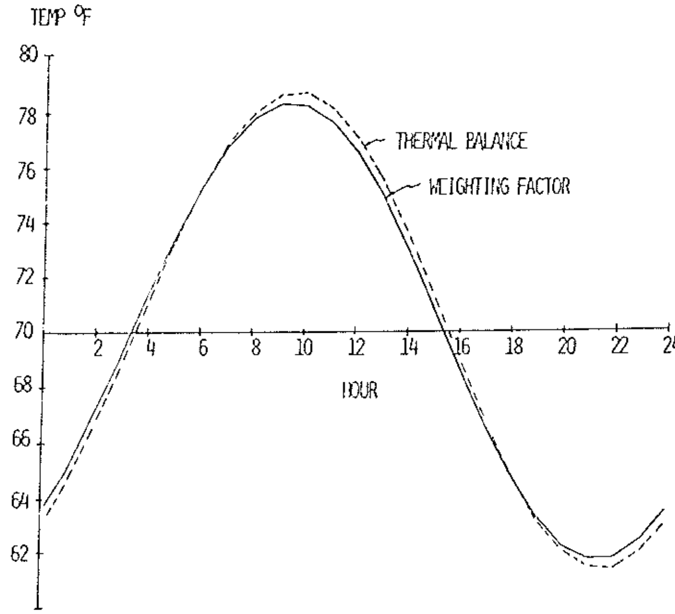


Fig. 4.

Weighting Factor/Thermal Balance Space Air Temperature Response for a 10K BTU Sinusoidal Radiation Input and a 10°F Sinusoidal Outside Air Temperature Input

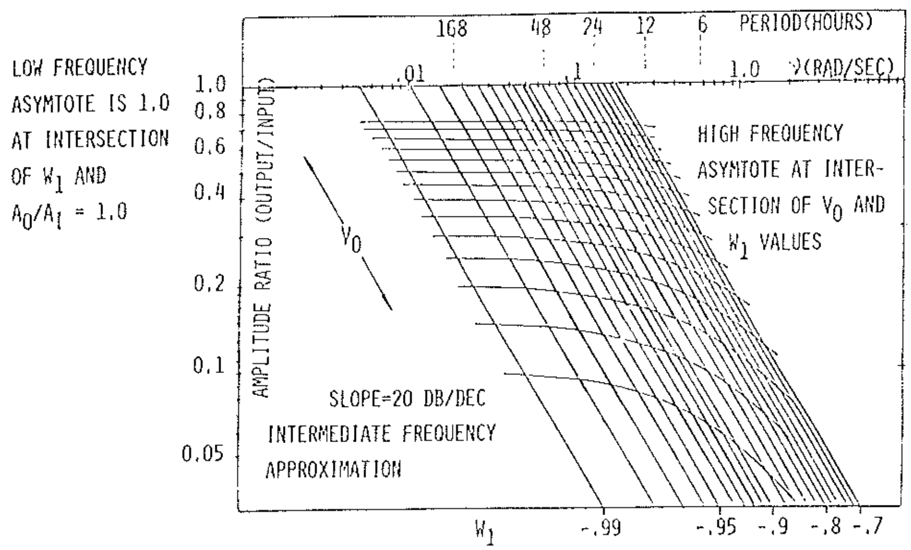


Fig. 5.

Composite Amplitude Ratio

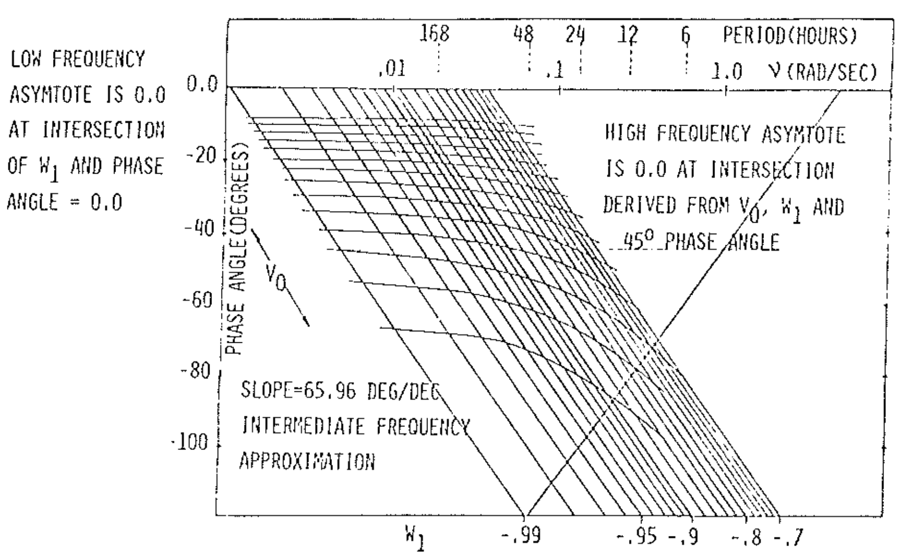


Fig. 6.

Composite Phase Angle

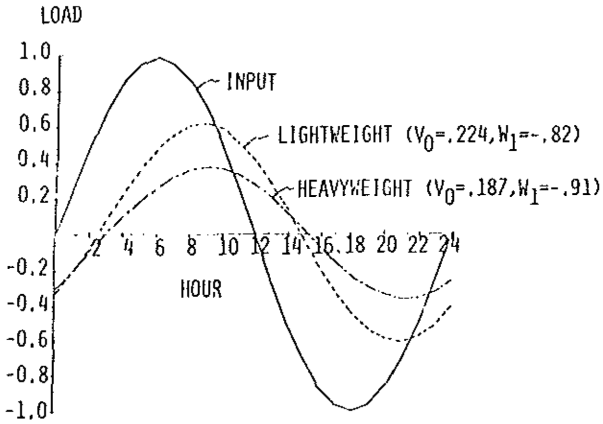


Fig. 7.

Response to a Unit Sinusoidal Radiation Input

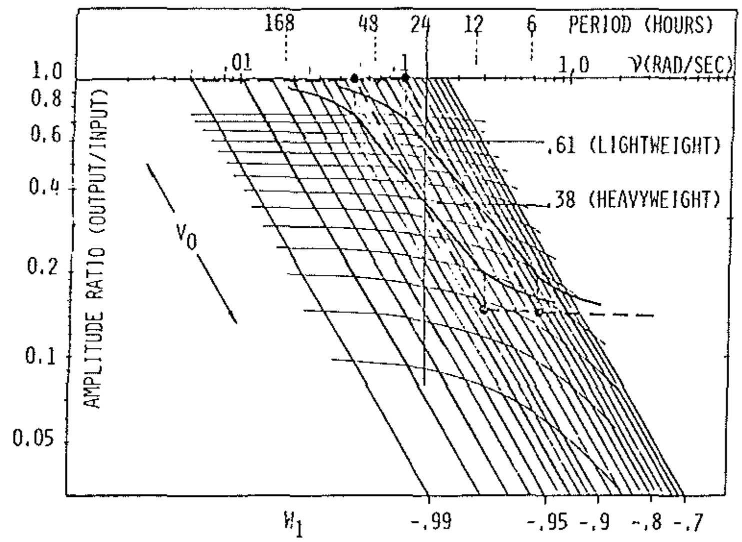


Fig. 8.

Application Technique Composite Amplitude Ratio Diagram

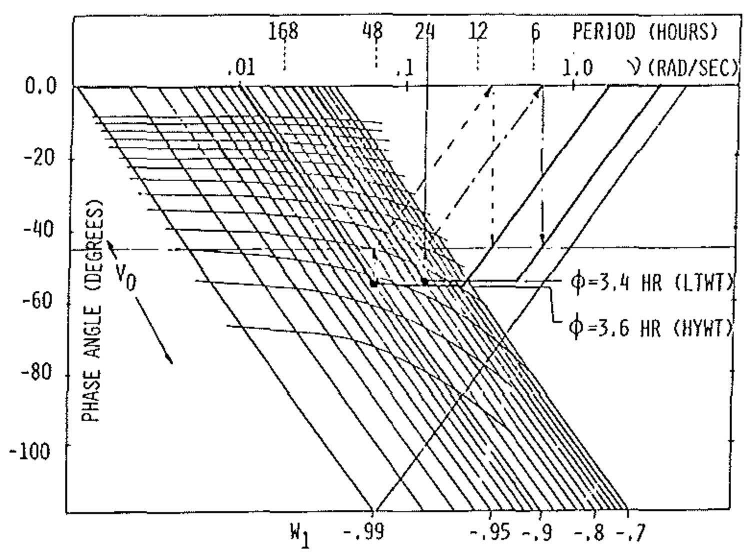


Fig. 9.

Application Technique Composite Phase Angle Diagram